EXAMPLE 6 Finding an Exponential Growth Rate

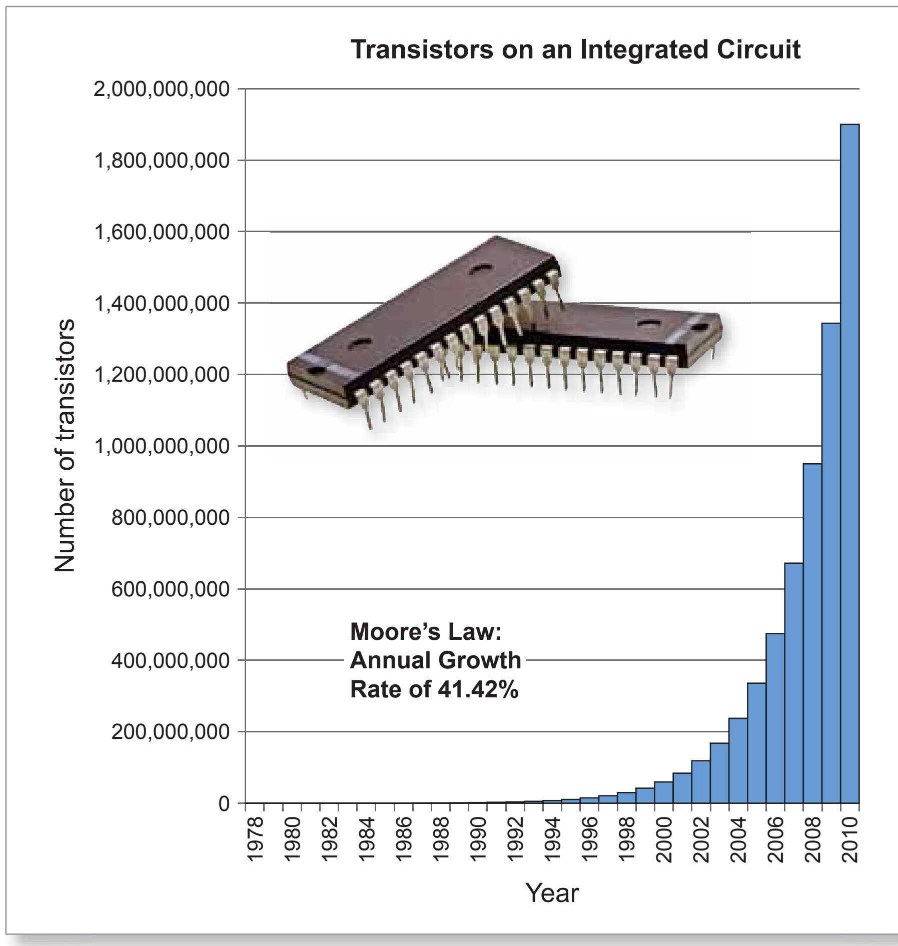
Moore's Law states that the number of transistors that can be placed inexpensively on an integrated circuit will double every *two years* (see Example 4 on page 155). What is the *annual* rate of growth represented by Moore's Law?

SOLUTION

As it turns out, the answer to this question has a fascinating history in mathematics. The question comes down to this: *Can you find a number whose square is 2?*

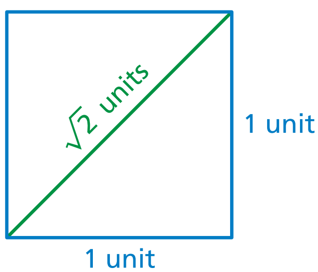
Square Root of 2

The answer is denoted by $\sqrt{2}$, which is approximately 1.414213562. So, the annual rate of growth for Moore's Law is about 41.42%.



Notice that the graph is similar to the graph obtained in Example 4 on page 155.

The ancient Greeks were able to prove that the square root of 2 cannot be represented by a fraction like 707/500. They were also able to construct a line segment with a length of exactly $\sqrt{2}$ units. To do this, draw a square with side lengths of 1 unit. Using the Pythagorean Theorem, it follows that the length of each diagonal of the square is $\sqrt{2}$ units.



Study Tip

Here are the first 201 digits of the square root of 2.

1.4142135623 7309504880
1688724209 6980785696
7187537694 8073176679
7379907324 7846210703
8850387534 3276415727
3501384623 0912297024
9248360558 5073721264
4121497099 9358314132
2266592750 5592755799
9505011527 8206057147

✓ Checkpoint

Help at Math.andYOU.com

Using an initial value of 29,000 in 1978 and an annual growth rate of 41.42%, how many transistors could be placed on an integrated circuit in 2010?