

Deductive Reasoning Systems



Destruction of the Great Library of Alexandria

When Alexander the Great died in 323 B.C., his empire was divided into different regions. Ptolemy I, one of Alexander's generals, took control of Egypt. Euclid lived in the cosmopolitan city of Alexandria and had access to libraries and other mathematicians. The royal library in Alexandria is thought to have been the largest and most comprehensive library in the ancient world. Its destruction is considered to be one of the greatest losses of knowledge ever experienced by humanity.

Unlike biology, chemistry, and physics, mathematical systems are not “discovered.” Each of the logical systems within mathematics was invented and developed by humans. For instance, the logical system called Euclidean geometry was invented and developed by the Greek mathematician Euclid around 300 B.C.

It is meaningless to talk about whether a mathematical system is “true.” No one can prove that Euclidean geometry is true. What you can do is decide whether the system is logically consistent, *assuming its premises are true*.

A logical system consists of four basic types of statements and concepts.

- Undefined terms
- Defined terms
- Unproven postulates (statements)
- Proven theorems (statements)

EXAMPLE 3 Analyzing a Logical System

Euclid understood that “you can’t prove everything.” At some point in developing a logical system, you have to use words that are not defined and you have to use premises that are not proven. For instance, Euclidean geometry does not define the concepts of “point” or “line.” You just assume that you know what they are. Here are the five premises (or postulates) on which Euclidean geometry is based.

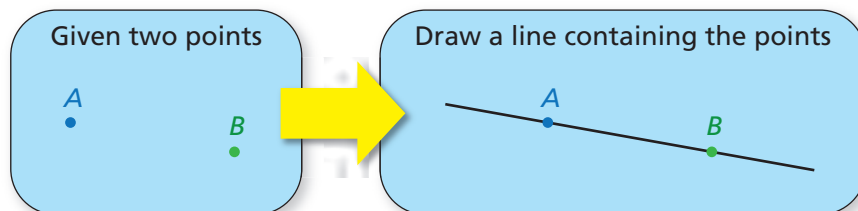
1. A unique straight line can be drawn between any two points.
2. A straight line segment can be extended to any finite length.
3. A circle can be described with any given point as its center and any distance as its radius.
4. All right angles are equal.
5. **Parallel Postulate:** At most, one line can be drawn parallel to a given line through a given point not on this line.

Write a syllogism that involves Euclid’s first postulate and illustrate it.

SOLUTION

Here is one way to write a syllogism that involves Euclid’s first postulate.

- **Premise:** Given any two points, there is a unique line that contains them.
- **Premise:** You are given points A and B .
- **Conclusion:** There exists a unique line containing A and B .



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Write a syllogism that involves Euclid’s fifth postulate and illustrate it.