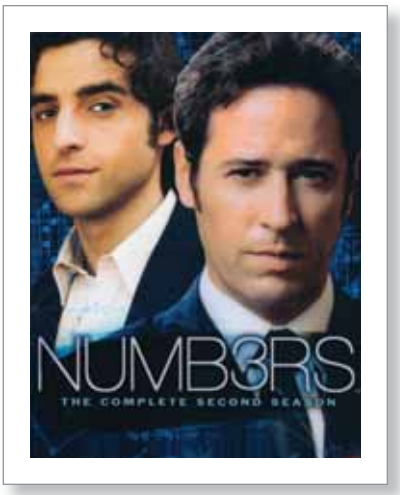
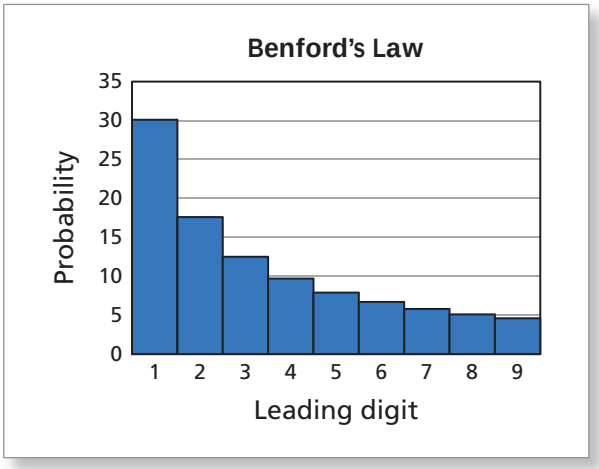


Benford's law was used by the character Charlie Eppes to help solve a case in the "Running Man" episode of the television crime drama *NUMB3RS*.

EXAMPLE 2 Using Independent Events to Detect Fraud

Benford's law states that in real-life data, the leading digit is 1 almost one-third of the time. Digits greater than 1 occur as the leading digit with decreasing frequencies, as shown in the table.

Leading digit	Probability
1	30.1%
2	17.6%
3	12.5%
4	9.7%
5	7.9%
6	6.7%
7	5.8%
8	5.1%
9	4.6%



You work for the IRS. As a means of randomly selecting tax returns to audit, you propose the following. A computer randomly selects one of the amounts from the first page and a second amount from the second page. If both amounts have 9 as a leading digit (as in \$93.28 and \$901.92), the return will be audited.

a. Using Benford's law, what percent of the returns should be audited?

b. Suppose each digit is equally likely to occur as a leading digit. What percent should be audited?

SOLUTION

a. According to Benford's law, you should expect to select 9 as the leading digit only 4.6% of the time. The probability of selecting two 9s is

$$\text{Probability of selecting two 9s} = (0.046)(0.046) \approx 0.00212.$$

So, about 0.2% of the returns (about 2 out of 1000) should be audited.

b. If each digit is equally likely to occur as a leading digit, then a 9 should be selected as the leading digit one-ninth of the time. The probability of selecting two 9s is

$$\text{Probability of selecting two 9s} = \left(\frac{1}{9}\right)\left(\frac{1}{9}\right) \approx 0.012.$$

So, about 1.2% of the returns (about 12 out of 1000) should be audited.

Checkpoint

Help at Math.andYOU.com

The following excerpt from *The New York Times* indicates that Benford's law can be used to spot fraudulent tax returns. Explain how this could happen.

> "Dr. Hill is one of a growing number of statisticians, accountants and mathematicians who are convinced that an astonishing mathematical theorem known as Benford's Law is a powerful . . . tool for pointing suspicion at frauds, embezzlers, tax evaders, sloppy accountants . . .."