



In spherical geometry, the undefined term *line* is imagined to be a great circle of a sphere. Longitude lines on Earth are great circles. Latitude lines, except for the equator, are not great circles.

EXAMPLE 4 Comparing Logical Systems

There are many possible “non-Euclidean” geometries. One is called spherical geometry. The difference between Euclidean and spherical geometry lies in what you assume the undefined term *line* to be and also in the Parallel Postulate. Here is one set of postulates for spherical geometry.

1. A unique straight line can be drawn between any two points, unless the points are antipodal, in which case they lie on many straight lines.
2. Any straight line segment can be extended indefinitely in a straight line. (At some point, it will connect with itself, but it can go around the sphere an infinite number of times.)
3. A circle can be described with any given point as its center and any distance as its radius, as long as the radius is less than half the circumference of the sphere.
4. All right angles are equal.
5. **Parallel Postulate:** No two lines are parallel. (Any two intersect.)

Explain why the Spherical Parallel Postulate makes sense.

SOLUTION

If you were to tell someone that “no two lines are parallel,” that person might think that you are irrational. What needs to be understood is that your understanding of terms is different.

- In spherical geometry, you understand a “line” to be a great circle of a sphere. For you, a “plane” is the surface of the sphere.
- In Euclidean geometry, you understand a “line” to be straight, like a laser beam. For you, a “plane” is a flat surface.
- You both understand that two lines are “parallel” if they lie in the same plane and don’t intersect.

To see why there are no parallel lines in spherical geometry, imagine putting a rubber band around a tennis ball. Make it a great circle so that it has a maximum circumference. Now try to put a second rubber band (great circle) on the ball. You can’t do it without intersecting the first rubber band.



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In Euclidean geometry, you know that the sides of a triangle are three straight line segments. You also know that the 3 angle measures total 180° . Sketch and describe a triangle in spherical geometry. What can you say about the sum of the angle measures of a triangle in spherical geometry?